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Discussion of
"MORNING-GLORY SHAFT SPILLWAYS:
DETERMINATION OF PRESSURE-CONTROLLED PROFILES"

by W. E. Wagner
(Proc. Sep. 432)

W. E. WAGNER.*—The writer is indebted to the individuals who devoted so much of their time and energy in preparing their constructive discussions of the paper. The discussions are valuable contributions to the problem of properly designing the overflow section of a shaft spillway.

The graphs, prepared by Messrs. White and McPherson and supported by experimental data, clearly define the region of low flow in which "scale effects" must be considered. It is interesting that weirs having heads as low as 0.10 foot (figures 17 and 19) may be used without surface tension and/or viscosity affecting the discharge. It should be noted that these results were obtained in the laboratory under ideal conditions where close control of the approach conditions, sharpness and cleanliness of the weir, and steadiness of flow were maintained. However, in general practice, where ideal conditions are difficult to attain, the minimum head for flow over a weir should be somewhat greater, possibly as high as 0.20 foot, to assure flow conditions in which "scale effects" can be safely ignored.

The discharge formula, $Q = 1.66 \sqrt{g}\, D^{1.04} H_s^{1.46}$, developed by Messrs. White and McPherson for heads greater than 0.10 foot and $\frac{H_s}{R}$ ratios less than 0.60, is a convenient method of expressing the discharge in the range of free flow. Discharge coefficients computed using the above formula check within 1-1/2 per cent of the coefficients shown on Figure 9 for $\frac{H_s}{R}$ ratios between 0.20 and 0.50. For $\frac{H_s}{R}$ ratios greater than 0.50, the deviation increases, reaching over 4 per cent for $\frac{H_s}{R} = 0.60$, Figure 25. This deviation leads to the question of the upper limit at which weir flow changes in character to flow through a re-entrant tube. Messrs. White and McPherson showed graphically in Figure 18 that this change occurs at $\frac{H_s}{R} = 0.60$. It is difficult for the writer to visualize an abrupt change from weir to tube flow as indicated in Figure 18 for the following reasons:

1. As pointed out by Mr. Blaisdell in his discussion, the head on a weir is usually considered to be affected when the "tail water reaches a level above the weir crest exceeding the critical depth of flow d_c," or when $\frac{Y}{H_s} = 0.667$. Table 4 shows that this tail water (or boil height) is reached when $\frac{H_s}{R}$ is between 0.45 and 0.50. When $\frac{H_s}{R} = 0.60$, $\frac{Y}{H_s} = 0.94$ or the high point of the boil is at a level above the crest equal to 0.94 of the head. A backwater of this

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height surely affects the head upstream from the weir and causes a departure from true weir flow at some $\frac{H_s}{R}$ ratio less than 0.60. (See Figure 26 which shows the operation of the circular weir when $\frac{H_s}{R} = 0.50$).

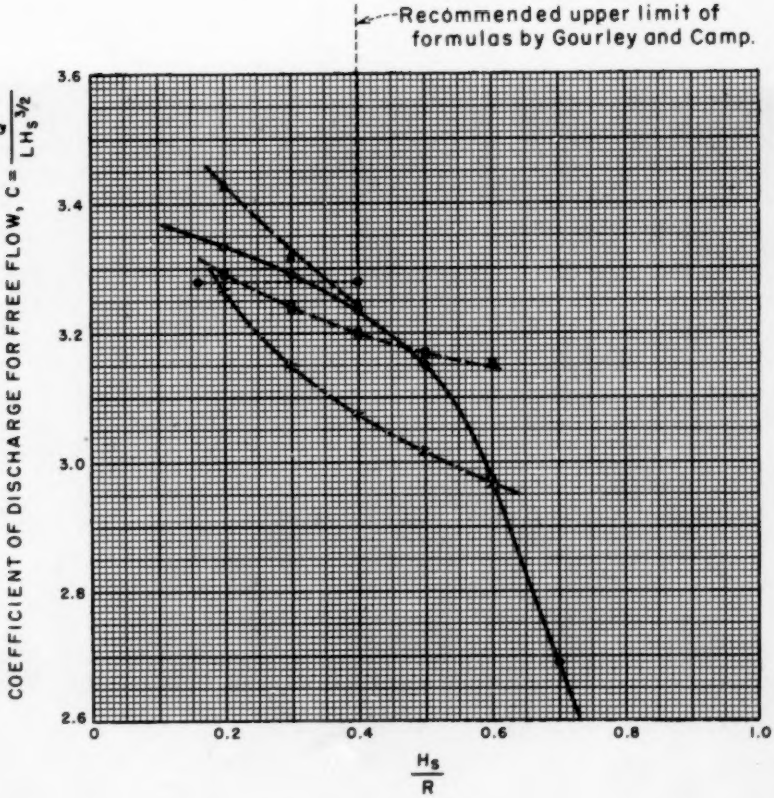
2. In Figure 27, some of the writer's original data are plotted on a graph similar to Figure 18. This plot clearly indicates that a transition zone exists between tube and weir flow and supports the original conclusion that the upper limit of weir flow is $\frac{H_s}{R} = 0.45$ and the lower limit of tube (or "orifice") flow is $\frac{H_s}{R} = 1.00$. Unfortunately, actual calibration points by other experimenters are not available. However, the published reports by both Camp(2) and Gourley(3) indicate a similar transition zone since they recommend an upper limit of $\frac{H_s}{R} = 0.40$ when using their formulas.

The writer questions whether a vortex can form in the range of flows where $\frac{H_s}{R}$ is less than 0.60 as proposed by Messrs. White and McPherson. At these flows, the velocity of the upper nappe surface is comparatively high and well directed with a clear line of demarcation between the boil and the nappe, as shown in Figure 26, where $\frac{H_s}{R} = 0.50$. Any vortex having a tendency to form in the nappe is carried immediately through the weir. Due to the turbulence within the boil, it is unlikely a vortex can form and maintain itself in this region. However, air may become entrained where the nappe joins the boil and perhaps affect the weir discharge, but it is questionable whether radial piers or baffles will prevent the entrainment of air. C. J. Posey and H. Hsu(9) found in their vortex studies that "with purely radial inflow the vortex is small and transitory; its effect on discharge is negligible."

Mr. Blaisdell questions the writer's statement "that most morning-glory spillways are designed for free flow." Messrs. White and McPherson in their discussion state, "Of the 18 morning-glory spillways for which details have been published(3) only 4 are designed to perform submerged, 6 for ratios of $\frac{H_s}{R}$ between 0.20 and 0.40, and 8 for ratios of $\frac{H_s}{R}$ less than 0.20." With over 75 per cent of the reported morning-glory spillways designed to operate in the free-flow range, the statement can hardly be questioned.

As suggested by Mr. Blaisdell, the discharge coefficient, C_o, in $Q = C_o A_o \sqrt{2gHs}$, for $\frac{H_s}{R}$ values between 0.4 and 2.0, is given in Figure 28. For $\frac{H_s}{R}$ values greater than 1.00, the coefficient is practically constant at 0.51. This value checks very closely the coefficient of 0.52 commonly used for flow through a re-entrant tube.(10) For comparison, the analytical results by Messrs. Elassiouty, Chanda, and Johnson expressed in terms of C_o and $\frac{H_s}{R}$ are also shown in Figure 28. The two results compare remarkably well, considering the entirely different methods employed in determining the coefficients.

The comparison by Messrs. Elassiouty, Chanda, and Johnson of the profiles measured by Camp and those by the author in Figure 23 lends support to the reliability of the nappe shapes. It is particularly gratifying that the profiles check in the vicinity of the weir spring-point, since this region is important and extremely difficult to measure with accuracy.



o C = Variable (Wagner)

□ $C = \frac{1.66}{\pi} \left(\frac{H_s}{D}\right)^{-.04} \sqrt{g}$ (Lehigh U.)

Δ $C = \frac{2.97}{H_s^{0.08}}$ (Gourley)

x $C = \frac{2.79}{H_s^{0.088}}$ (du Pont)

• C = 3.28 (Camp)

FIGURE 25 – COMPARISON OF FREE-FLOW DISCHARGE COEFFICIENTS DETERMINED BY VARIOUS EXPERIMENTERS (NEGLIGIBLE APPROACH VELOCITY AND AERATED NAPPE)

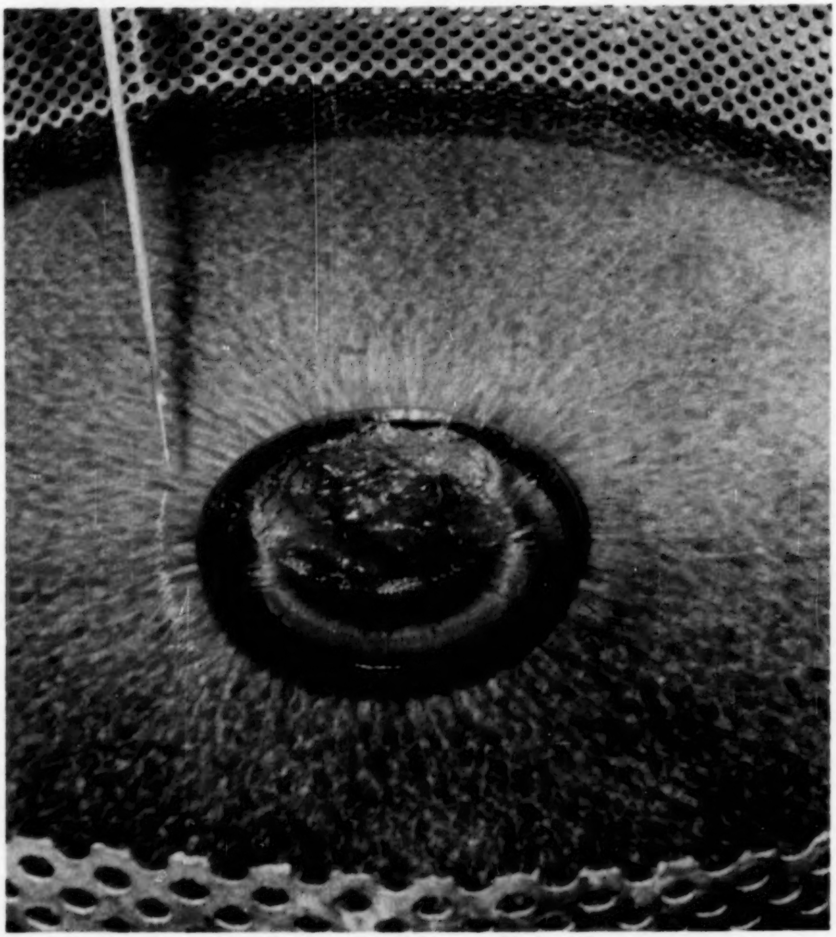
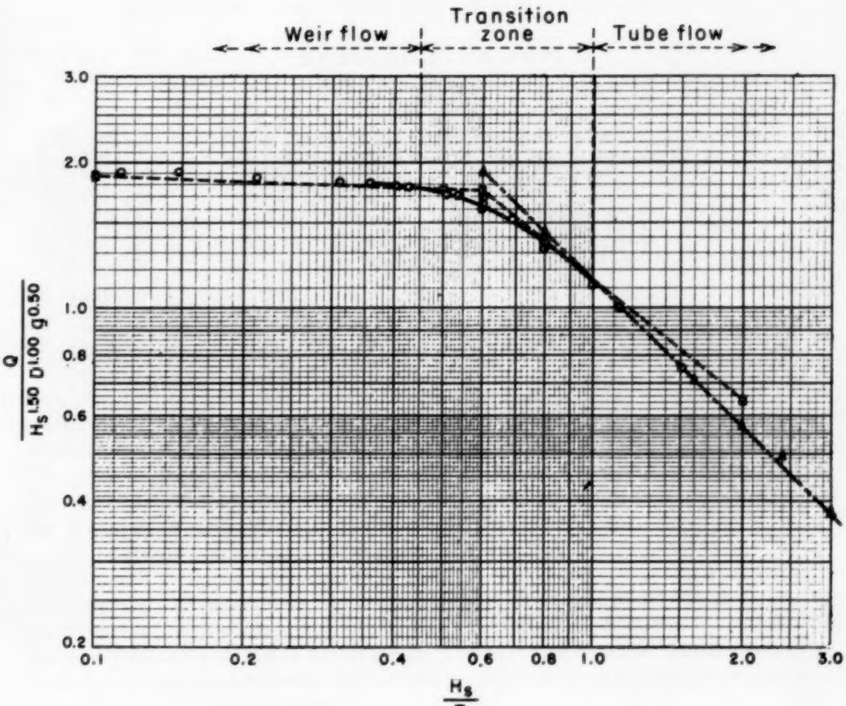


FIGURE 26 - FLOW OVER CIRCULAR WEIR WHEN $\frac{H_s}{R} = 0.50$



o——— Wagner experimental data.

Δ—-— Re-entrant tube formula, where $C_0 = 0.513$.

□---- Lehigh U. data from Figure 18.

FIGURE 27 – COMPARISON OF LEHIGH AND WAGNER DATA IN REGION WHERE WEIR FLOW CHANGES TO TUBE FLOW

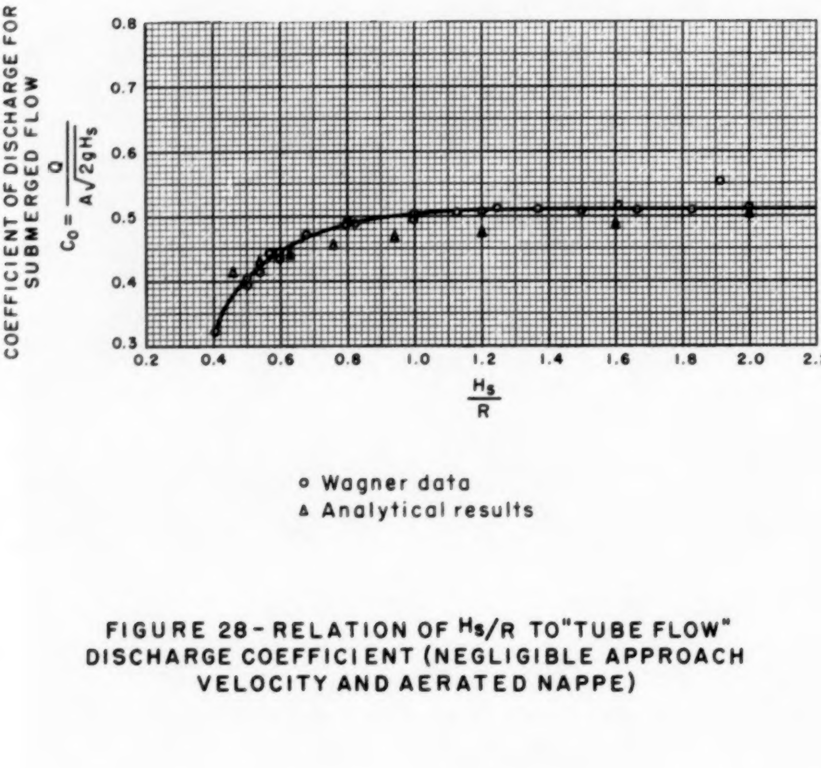


FIGURE 28 - RELATION OF H_s/R TO "TUBE FLOW" DISCHARGE COEFFICIENT (NEGLIGIBLE APPROACH VELOCITY AND AERATED NAPPE)

Discussion of
"DIVERSION FLOW THROUGH BUFORD DAM CONDUITS"

By Francis F. Escoffier
(Proc. Sep. 535)

HAROLD TULTS.*—The graphical method, developed by Mr. Francis F. Escoffier, is of great help in computing the water surface profiles in steady non-uniform flow, particularly in the open conduits of variable cross-section.

The described application of this method to establish the reservoir levels, at which the flow discharge pulsates between the open-conduit-flow and the pressure-conduit-flow, is very instructive. However, the writer questions whether the predicted range of the reservoir levels, at which the pulsations occur, is confirmed by the actual measurements at the erected diversion conduits.

There are several other factors affecting the sensitive transition of the flow in the diversion conduits. Besides the simplifications, made by the author, the following may affect the transition of the flow: (a) The inaccuracy of the graphical method of determining the critical depths and the minimum energy heads; (b) the effect of the curved flow on the pressure distribution in the intake transition; and (c) the variable n-value of the friction coefficient in the Manning formula for the part-full conduit flow.

(a) To establish a point of tangency, presenting the critical depth for a certain energy level, especially at the flat F-curves, is never accurate. Considering the amount of the mathematical work required for the plotting of the F-curves of the circular and of the rectangular sections, it seems to be more expedient and more accurate to compute the critical depths and the corresponding minimum energy levels analytically, employing the general equation for the critical flow

$$\frac{Q^2}{g} = \frac{a^3}{C_v b} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. 1}$$

C_v coefficient of velocity (Coriolis)
b width of water surface
All other symbols, if not defined, have the same definition as given by the author.

This Equation is adaptable for any open conduits, whatever cross-sectional shape it has. For a part-full circular conduit, the critical depths and the energy levels are easily derived by the Eq. 2 adapted for the circular cross-section,

$$Q = \sqrt{\frac{g}{C_v} \quad \frac{a^3}{2\sqrt{Dd_c - d_c^2}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. 2}$$

in which the area, a, of the wetted section is:

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$$a = \frac{\pi}{8} D^2 + (d_c - \frac{D}{2}) \sqrt{Dd_c - d_c^2} + \frac{D^2}{4} \sin^{-1} \frac{2d_c - D}{D} \quad \ldots\ldots\ldots\ldots \text{Eq. 3}$$

When $d_c < \frac{D}{2}$, the last two terms in Eq. 3 are negative.

(b) The water level in an air vent, located at or near a curved boundary, is not determinable by the straight-line distribution of the static pressure. The flow of the air through the vent into the closed conduit may continue due to the locally reduced pressure in the curvilinear flow. Thus, a higher reservoir level than that found by the author's graphical method, would be required to prime the circular conduit. Of course, the pressure deviations may be established by drawing of the flow nets.

(c) The application of the author's graphical method in determining the water-surface profiles in the part-full closed conduits requires a modification of the conventional friction coefficient, which varies from the one established for the full conduit.

For many years engineers(1,2,3,4) have found that using the friction coefficients, established for the pressure conduits, in the computations of the flow in the part-full conduits, yield incorrect results. Charles W. Sherman,(5) analysing the tests by O. Wilcox, found when employing the conventional hydraulic radius that the friction coefficient, established for the full-pipe flow, should be increased up to 135 per cent for the part-full pipe. This contradicts the still wide-spread assumption that the conveyance capacity of a closed conduit is maximum when partly full, (for circular conduit at $d = 0.93D$).

This may be explained by the air friction. Although not demonstrable in the open conduits, the retarding effect of the air friction in the part-full closed conduits is obviously increased by the closeness of the conduit ceiling.

To take care of the air friction over the water surface in the part-full circular conduit, Prof. A. Schocklitsch(6) includes the width, b, of the free water surface to the wetted perimeter, p, after reducing it to an equivalent wetted width, αb, of the same roughness as the actually wetted conduit wall. Thus, the total wetted perimeter of the part-full conduit is

$$p_{eff.} = p + \alpha b \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. 4}$$

Employing the same friction coefficient n (Manning) as for full conduit, the ratio between the mean velocities at part-full and full-pipe flow is:

$$\frac{V}{V_{Full}} = \frac{\frac{1.486}{n} S^{0.5} (\zeta \frac{D}{4})^{2/3}}{\frac{1.486}{n} S^{0.5} (\frac{D}{4})^{2/3}} = \zeta^{2/3} \quad \ldots\ldots\ldots\ldots \text{Eq. 5}$$

The quantity of ζ is expressed as follows (for symbols see Fig. 1):

$$\zeta = \frac{\frac{\psi\pi}{180} - \sin\psi}{\frac{\psi\pi}{180} + 2\alpha\sin\frac{\psi}{2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{Eq. 6}$$

In Fig. 2, the curves of the velocity ratio, $\frac{V}{V_{Full}} = \zeta^{2/3}$, for the reduction coefficient, α, equal to 2, 1, 0.4, 0, and -0.4 are plotted.

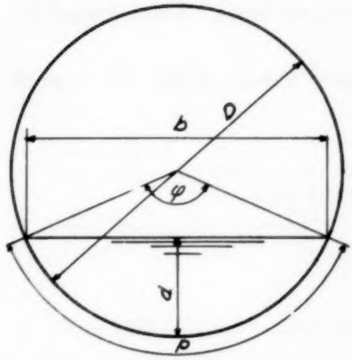

Fig.1 -Part-full circular conduit

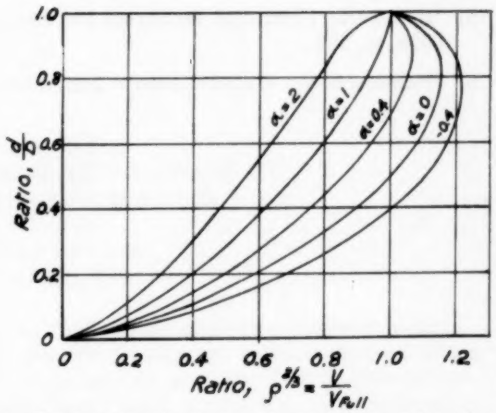

Fig.2-Velocity Ratio for various depths and reduction coefficients, α

The curve, $\alpha = 0$, represents the variation of the velocity ratio at different filling depths when the same friction coefficient as for the full conduit is also applied for the part-full conduit, as practiced currently.

Prof. Schocklitsch, having investigated the discharge capacities of various part-full circular conduits, recommends that at normal conditions when no forced air movements in the conduit exist, a coefficient α equal to 0.4 should be used.

When, as a result of external atmospheric conditions, an air-movement is induced in the conduit, the coefficient may be greater or smaller than 0.4, depending on the direction of the air flow. α equal to zero is justified, when the air movement is in the flow direction and of the same velocity as the flowing water.

As shown, the application of the author's graphical method, excellent in open-conduit-flow, meets extra complications in determining the range of the pulsations at the transition from the open to the pressure flow and reverse, and also in establishing the water-surface profiles at part-full flow.

In the solution of such delicate problems, like the one presented by the author, a combination of his graphical method with analytical computations may be helpful to increase the accuracy of the result.

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Discussion of
"THE LOG-PROBABILITY LAW AND ITS
ENGINEERING APPLICATIONS"

by Ven Te Chow
(Proc. Sep. 536)

SHIGEHISA IWAI.*—The paper by Prof. Chow should be highly evaluated in presenting quite an interesting procedure to revise Hazen's Method with a more sound theory obtained after re-examining its basic distribution which originally follows the log-probability law. Here, as one who is interested in such a research, the writer wishes to state his opinion about it as follows.

In order to apply the log-probability law widely to the hydrologic data which fails in satisfying Eq. 16, we generally have to increase the number of parameters in addition to the two contained in the basic Equations 5 and 7. Prof. Chow happened to perform this treatment by adopting, not the computed value of C_s, but its graphically determined value.

For the sake of such a treatment, we can, in Eq. 7, replace the reduced variate $y = \log_e x$ by a new one represented by any of the following three forms:

$$y = \log_e (x + b) \tag{i}$$

$$y = \log_e \left(\frac{x + b}{g - x}\right) \tag{ii}$$

or as a simplified form of the above in which $b = 0$,

$$y = \log_e \left(\frac{x}{g - x}\right) \tag{iii}$$

where (-b) or g is a constant corresponding to the lower or the upper limit of the variate x, respectively, in the original frequency function $\phi(x)$. Therefore, each of Eqs. (i), (ii) and (iii) represents the distribution in which the value of x is restricted within the range from (-b) to $(+\infty)$, or from (-b) to g, or from 0 to g, respectively. Also it is noted that Eqs. (i) and (iii) have one more additional parameter, while Eq. (ii) has two.

When we take the new y by Eq. (i) and interpret $\bar{y}$, σ_y and K_y on the basis of this y, equations corresponding to Eqs. 11 and 14 are obtained:

$$\bar{x} = e^{\bar{y} + \sigma_y^2/2} - b \tag{iv}$$

$$C_v = F'(e^{\sigma_y^2} - 1)^{1/2} \tag{v}$$

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where F

$$F' = \left(1 + \frac{b}{\bar{x}} \right) \qquad \text{(vi)}$$

can be called as "a correction factor for the coefficient of variation C_v" similar to the form of F by Eq. 43 for C_s.**

Furthermore, it is quite interesting that Eq. 15 for C_s does not change its form in this case and, therefore, we can obtain the following relation between C_s, C_v and F' instead of Eq. 16.

$$C_s = 3\left(\frac{C_v}{F'}\right) + \left(\frac{C_v}{F'}\right)^3 \qquad \text{(vii)}$$

Meanwhile, since the frequency factor K in Eq. 28 still maintains the same form as represented by Eq. 29, Prof. Chow's principle to perform straight-line fitting through K is also valid for the new y. By utilizing the equation,

$$\sigma_y = \sqrt{\log_e \left[1 + \left(\frac{C_v}{F'}\right)^2\right]} \qquad \text{(viii)}$$

which is given from Eq. (v), the value of σ_y can be readily calculated for a fixed value of C_v, and for any of the assumed values of either $\frac{b}{\bar{x}}$ or F'. Thus, the obtained σ_y enables us to compute K with various K_y corresponding to any probability P. By comparison with Professor Chow's procedure which requires a rather tedious calculation of σ_y by C_s according to Eq. 34, the afore-mentioned procedure is quite simple in preparing similar tables to Table 2 and 4, and eventually, in constructing the chart and the probability paper being alike to Fig. 2 and 3. Finally, in this case, all of the values indexed by C_s in these figures should be substituted by the corresponding $\frac{b}{\bar{x}}$ — or F', while the fitting procedure remains quite the same as the author's.

The factor of F' can be utilized as an index to measure quantitatively and qualitatively the discrepancies between any estimated distribution and the standard one by Eqs. 5 and 7. This standard distribution should be shown as a straight line with an index of F' = 1 on Fig. 2. As the author stated, the accuracy of computed value of C_s depends greatly upon the number of items in the series used for computing it. In this sense, the computation of C_s by Eq. 39, proposed again in step 1 by the author, would not be essentially needed. However, should we want to compare the computed C_s with its theoretical value, Eq. (vii) can easily give the value.

For the distribution by Eq. (i), a solution by the method of moments was presented by Slade.(1') Also, a procedure to fit a straight line, after semi-graphical trial caluclations, on the basis of y, was already proposed by Gibrat.(40) While Prof. Chow's results in this paper would be much more remarkable in performing the straight line fitting on the basis of the original

** However, for an actual adjustment enabling us to hold the relation of Eq. 16, as shown by Eq. (vii), the reciprocal $(\frac{1}{F'})$ should be used to multiply the C_v in Eq. (v), that is computed in step 1 proposed by Hazen and Prof. Chow.

variate x, by devising a special probability paper, it should be noted that the above criticism by the writer is just limited on author's process before he obtained the results. The laborious effort by the author in preparing the tables and the figures, mentioned above, must be highly appreciated. Since the F' and the C_s in Eq. (vii) hold a one to one correspondence at a fixed C_v, these tables and figures will never lose their practical values.

For the distribution by Eq. (ii), and eventually by Eq. (iii), the general formula of the r-th moment M_r was accurately obtained by the writer,(42) which would be probably used for such a straight line fitting. For the distribution, especially by Eq. (iii), the writer has developed several practical procedures.(2')(3')(4')

By increasing the number of parameters, we can automatically improve the flexibility of the curve-fitting. Therefore, it is no wonder that C_s of the extreme-value law having two parameters, is fixed at a value of 1.139 under varying C_v; but distributions having more than two parameters, can produce various combinations of C_s and C_v. However, regarding the practical flexibility of the extreme-value law, the writer has the same opinion as Gumbel.(56) Generally too many parameters cause a more difficult theoretical solution and also, when we determine them by graphical inspection, too many combinations rather harm our quick decision on it.

According to our experience in Japan, the extreme-value law can be applied in a considerably wide range of C_s near $C_s = 1$; and the law by Eq. (i) is applicable in an even wider range of C_s. For the distribution extremely skewed, Kadoya,(5') one of our research members, has proposed the "log-extreme value law" of which the basic equations are

$$1 - P = e^{(-e^{-\xi})} \tag{ix}$$

$$\xi = k\left[\log_e(x + b) - \log_e(x_0 + b)\right] \tag{x}$$

where ξ is a reduced variate, x_0 is the mode of x, and b, k are parameters. For practical usage, he has already tabulated the values of these parameters. It is quite interesting that this law was presented almost simultaneously with, but quite independently of, Gumbel's new proposal.(6')

Practically, the author's procedure in this paper would be regarded almost the same as the conventional methods, requiring the troublesome computation of moments, though it might be of great value in the United States, where Hazen's Method is prevailing. We can, alternatively, take account of the method of maximum likelihood for the distribution by Eq. (i). In order to determine the three parameters contained in this distribution, a solution of complicated transcendental equations should be performed. However, Nishihara,(7') another member of our group, has obtained very useful formulas, as a result of the solution after linearizing the equations.

The writer wishes to express his highest respect for Prof. Chow's results obtained in his paper with many thanks for his kind correspondence inviting the writer to take part in this discussion. It is truly hoped that the further exchange of information in this field will be carried out between the researchers in various countries.

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R. D. GOODRICH,** M. ASCE.—For more than twenty-five years the writer has found that the logarithmic transformation has been "a convenient tool for recording, fitting, and frequency analysis of various engineering data." Hence this paper has been studied with great interest. It serves to fill a yawning gap by presenting so ably the mathematical foundations and the logical developments which will make the engineering applications of the log-probability law more numerous and more useful.

It is understood of course that the arithmetic mean is the most probable and basic value when statistical data is analyzed and assumed to be distributed according to the normal law. When the logarithms are substituted for the quantities instead, their mean is the log. of the geometric mean of the quantities making up the original data. For the benefit of the large number of engineers whose mathematical training does not extend to the heights demonstrated by the author, of which the writer is one, but who might wish to make more intelligent use of the material and methods given in this paper, it is suggested that the statement of the "central limit law" be amplified and more fully explained. Many of the references from which this information might be secured are not readily available except in a large university or other research library.

H. ALDEN FOSTER,*** M. ASCE.—The writer has been interested in the use of log-probability plotting of engineering data for many years, going back to the introduction of this method for study of Flood Flows by Allen Hazen in 1930.(39) In a discussion of Lane and Lei's paper (50) in 1950, the writer pointed out the importance of having some method to determine the

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median (M) when the mean value of x ($\bar{x}$) was known, or vice-versa. Not having any mathematical expression for this ratio as a function of I_v, he computed the mean by arithmetic integration of the frequency function of x, -Eq. 7. The values of x used in this calculation were read directly from the straight line plotted on the log-probability paper.

In 1952 the writer carried out additional studies of the log-probability function [51] in which the values of $\bar{x}/M$, C_v and C_s were determined for various values of I_v, again using an arithmetic-integration procedure since no mathematical formulas for these characteristics were available. Subsequent to this study, the author's mathematical analysis of the log-probability function was published.[56] When the writer compared his results for $\bar{x}/M$ with the values computed by the author's formula, it appeared that there was a considerable discrepancy in the results. A review of the mathematical analysis of [56] indicated an error in the formulas derived therein, as noted by the author in the footnote to Eq. 10.

The writer has reviewed the mathematical development of the present paper, and has verified the formulas up to Eq. 33, inclusive. As a further check of these formulas, he has made an arithmetic integration of the frequency curve for x, in order to obtain the characteristics $\bar{x}/M$, C_v and C_s, using values of x taken from a table of the probability integral. On account of the great magnitude of the x-values at very low probabilities, points were selected at close spacing for probabilities less than 1.0%, the minimum spacing being 0.002%. For the rest of the curve, the points were selected at the mid-points of each 2% on the "%-of-time" scale. The calculations were carried out for values of I_v equal to 1.00, 0.50 and 0.25. The results are summarized in Table A, in which are also shown the precise values of the same characteristics calculated by the author's formulas,—Equations 21, 14 and 16, respectively. The latter calculations were carried out for assumed values of I_v, and show good agreement with the corresponding results shown on the author's Fig. 1.

Examination of Table A shows that the arithmetic integration agrees closely with the mathematical formulas for $\bar{x}/M$ and C_v except for extreme values of I_v. The arithmetic value for C_s is also in close agreement, for $I_v = 0.25$; but for larger values of I_v there are appreciable discrepancies in the C_s results, due to the controlling effect of the extreme values of x. The writer believes that these calculations give a satisfactory empirical verification of the mathematical formulas.

Correction Factors for Computed Coefficients:

In the writer's paper on "Theoretical Frequency Curves" published in 1924,* an attempt was made to establish certain correction factors to be applied to the values of the Coefficient of Skew obtained from records of a limited number of items. The factors were obtained by plotting duration curves for certain assumed values of C_v and C_s, in accordance with the Pearson formula, Type III. The curves were plotted on arithmetic-probability paper; values of the ordinates were scaled from the diagram, for assumed sequences of 100, 20, 10 and 5 items; and the corresponding values of the C_v and C_s were computed. It was found that the C_v did not vary greatly with the length of the record, when the C_s did not exceed 0.60. On the other hand, there was an appreciable difference in the C_s with records of different lengths, and there would be a considerable error in the C_s even with a record of as many as 100 items.

* Trans. ASCE, Vol. 87, 1924, Pg. 142.

In a discussion of the paper referred to, Allen Hazen made the suggestion that the value of C_s computed from a record of n items could be multiplied by the factor, $F = 1 + 8.5/n$. Mr. Hazen made the same suggestion in his book "Flood Flows"(39) published in 1930; and this formula seems to have been rather extensively used since that time. The writer has felt for some time that Hazen's formula was not adequate, as it was evident that the error in the C_s would increase for large values of C_s as well as for smaller values of n. The log-probability method of plotting furnished a convenient means for revising the correction factors.

A series of values of the ordinates of a straight line plotted on the log-probability paper was obtained by use of the table of the probability function. The items were uniformly distributed on the %-of-time scale. 100 items were obtained, for 0.5%, 1.5%, 2.5%, etc; 50 items were used, located at 1%, 3%, 5%, etc; 20 items, at 2.5%, 7.5%, etc; and 10 items at 5%, 15%, etc. Three different probability curves were used, with I_v values of 0.50, 0.25 and 0.05 respectively. The results were analyzed, and plotted in Fig. A. Using the formulas

$$\text{Adjusted } C_v = \text{Computed } C_v\ (1 + F_v)$$
$$\text{Adjusted } C_s = \text{Computed } C_s\ (1 + F_s)$$

the values of F_v and F_s can be taken from the diagram, corresponding to values of n between 10 and 100, and for various values of the Computed C_v and C_s obtained from the recorded items.

It may be seen that the corrections for C_v are less than 10% if there are 50 or more items in the record and the computed C_v is less than about 1.0; but the corrections for C_s become quite appreciable for all cases except those where the computed C_s is quite small.

The curves of Fig. A were obtained by an interpolation process from a limited number of computed values; however they are believed to give an approximate indication of the corrections which should be applied to any series of items which approximate a straight line on the log-probability paper. They should also be suitable for all practical purposes for use with other types of skew-probability curves. In any case, they should give more reliable results than the Hazen formula.

Probability of the Mean Value:

One characteristic of the log-probability function has not been given by the author. It may be convenient to determine the %-of-time at which the mean ($\bar{x}$) of the x-values occurs on the probability curve. By Eq. 21,

$$\bar{x}/M = e^{1/2\,\sigma_y^2}$$

Since $y = \log_e x$: $x = e^y$. Hence, $\bar{x} = e^{y_m}$; and the median value of x, or M, = $e^{\bar{y}}$,- where $y_m = \log_e \bar{x}$.

Hence, $\bar{x}/M = e^{y_m}/e^{\bar{y}} = e^{1/2\sigma_y^2}$.

Or, $e^{(y_m - \bar{y})} = e^{1/2\,\sigma_y^2}$.

From this, $(y_m - \bar{y}) = 1/2\,\sigma_y^2$ $\qquad (y_m - \bar{y})/\sigma_y = 1/2\,\sigma_y$

Since $\sigma_y = 2.3026\ I_v$ (Eq. 22a),

$(y_m - \bar{y})/\sigma_y = 1.1513\ I_v$.

The probability integral tables give values of $(y_m - \bar{y})/\sigma_y$ vs. F(y).

Table A.

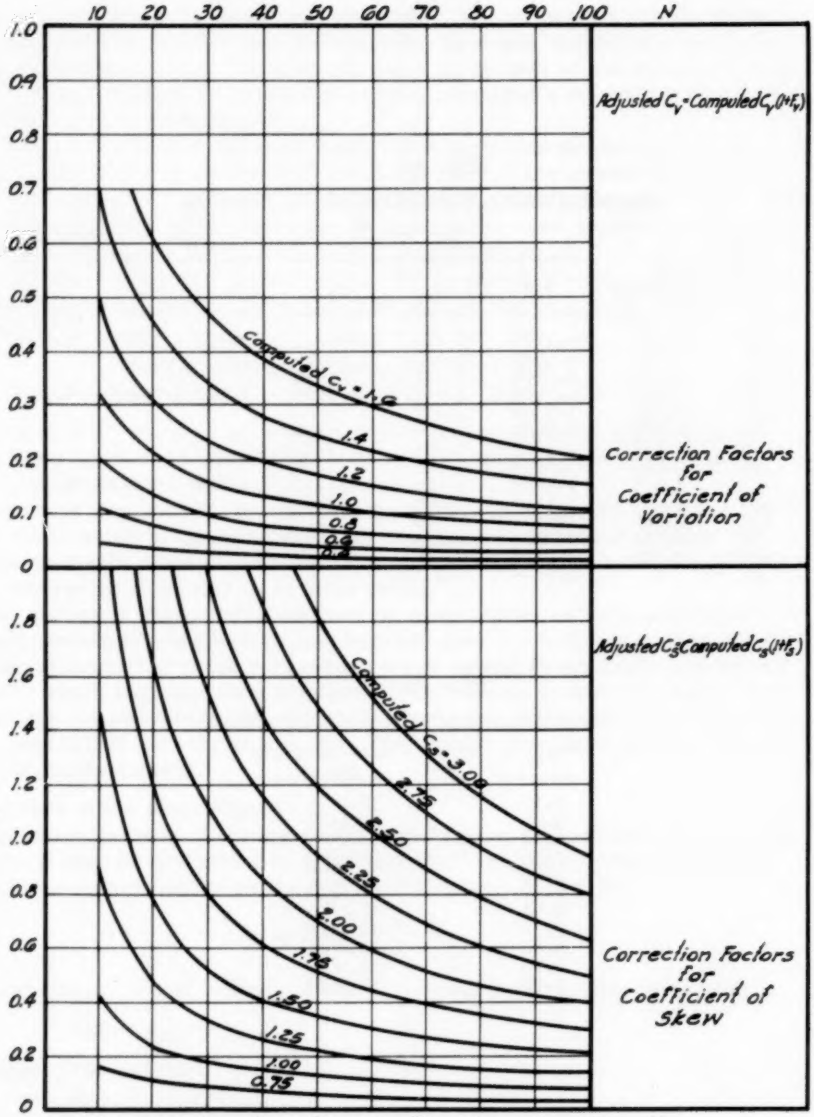
Characteristics of Log-Probability Function

I_v	$\sigma_y = 2.3026 I_v$	By Formula $\bar{x}/M$	C_v	C_s	Arith.-Integration $\bar{x}/M$	C_v	C_s
1.00	2.303	14.1675	13.970	2,965.	13.820	9.87	70.5
0.75	1.726	4.4423	4.328	94.0			
.60	1.381	2.5969	2.397	20.94			
.50	1.151	1.9401	1.663	9.59	1.927	1.645	8.40
.45	1.035	1.7106	1.388	6.829			
.40	0.920	1.5283	1.156	5.010			
.35	0.805	1.3837	0.956	3.743			
.30	0.690	1.2694	0.782	2.825			
.25	0.575	1.1802	0.626	2.124	1.179	0.626	2.10
.20	0.460	1.1119	0.486	1.573			
.15	0.345	1.0615	0.356	1.113			
.10	0.230	1.0269	0.233	0.712			
.05	0.115	1.0066	0.116	0.350			
0	0	1.0	0	0			

Table B.

Plotting Positions for Mean Value, $\bar{x}$.

I_v	%-Probability for $\bar{x}$.
0.00	50.0
0.10	45.4171
0.20	40.8966
0.30	36.4901
0.40	32.2572
0.50	28.2426
0.60	24.4852
0.70	21.0148
0.80	17.8515
0.90	15.0062
1.00	12.4805


10 20 30 40 50 60 70 80 90 100 N
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
F_V
Adjusted C_v = Computed $C_v (1+F_v)$
Computed C_v = 1.6
1.4
1.2
1.0
0.8
0.6
0.4
Correction Factors for Coefficient of Variation
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
F_S
Adjusted C_S = Computed $C_S (1+F_S)$
Computed C_S = 3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
1.00
0.75
Correction Factors for Coefficient of Skew
10 20 30 40 50 60 70 80 90 100 N
FIG. A

Hence it is possible to determine the %-of-time plotting position for y_m, which is also the plotting position for $\bar{x}$ for a given value of I_v. The computed values are shown in Table B.

In this paper the author has presented an excellent mathematical analysis of the logarithmico-probability function, and has outlined methods for its practical application to enginnering problems, which should prove valuable to the profession.

General

G. N. ALEXANDER, A.M. ASCE,[1] and A. KAROLY.[2]—For some years the writers have made use of the log-normal distribution in hydrologic analysis. The comprehensive treatment by the author of this distribution and its relation to the extreme-value distribution is particularly valuable as the statistical texts treat these distributions rather cursorily. The practical importance of the log-normal distribution is exemplified by the author's extensive bibliography. In hydrologic investigations the log-normal distribution has often been applied to flood flows (Author's refs. 36, 37, 39, etc.), although such data is also regarded as the legitimate field for extreme-value law. The log-normal distribution has been less frequently applied to the daily stream flow data in the form of flow-duration curves, although from the writers' experience on Victorian streams it is particularly applicable. It has therefore been used in practical applications such as estimating divertible flows for a given channel capacity, and for establishing criteria for stream flow accuracy. The relevant theory for these developments touches closely on the author's paper and hence will be given here, but the practical applications will not be discussed. The writers' discussion follows, in general, the sequence of development used by the author.

In the writers' Appendix I. new symbols are suggested for the more frequent terms, with the symbols used by the author alongside. In the text of the discussion the author's symbols are used, but in the writers' Figures the new symbols suggested are used.

Characteristic Relationships

A useful addition to the relations given by the author, which the writers derived from Equations 5 & 6 of Kalinske's paper (Author's Ref. 3) is

$$\left(\frac{\bar{X}}{M}\right)^2 = 1 + C_\sigma^2 = C_g^2$$

As the ratio of arithmetic and geometric means is an important parameter, it has been given a new symbol Cg.

In using commercial log-probability paper (as shown in the author's Fig. 2), the parameter obtained from the ratio of the ordinates at 50% and 15.87% (or 84.13%) is σg. This is therefore regarded as a more suitable

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) Water Supply
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) Australia.

parameter than σy for graphic representation of relationships, and is therefore used in the writers' Fig. 1. For determining the other parameters a series of values of Cv was assumed and Cg, σg, etc. calculated.

Flow Duration and Volume Distribution Curves

The flow-duration curve of the hydrologist shows the percentage of time (as abscissa) that the rate of flow (as ordinate) is greater or less than a given rate. However, for many applications we are concerned with the distribution of the volume of flow. This volume is the product of the flow rate x (in cusecs say) by the time during which such a rate is maintained.

When examining this problem the writers discovered that if the time-distribution $\psi(x)$ was a log-normal, the volume distribution was also log-normal, and what is more had the same standard deviation.

The frequency function (of the flow x) from equation 7[3] is -

$$\varphi(x) = \frac{1}{\sqrt{2\pi}\, e^{y}\sigma_y} e^{-\frac{1}{2}\left(\frac{y-\bar{y}}{\sigma_y}\right)^2}$$

The probability (of flow) less than the given variate is (Eq. 32) -

$$Q = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{K_y} e^{-\frac{1}{2}K_y^2} dk_y$$

where Q = 1-P and is shown graphically in the lower line of the writers' Fig. 2.

The frequency function of the volume distribution is x $\psi(x)$ and its probability, Q_1, of volume of flow less than the given variate can be shown to be given by

$$Q_1 = \frac{\bar{x}}{\sqrt{2\pi}} \int_{-\infty}^{K_{y_1}-\sigma_y} e^{-\frac{1}{2}(K_{y_1}-\sigma_y)^2} d(K_{y_1}-\sigma_y) \quad \text{where } K_{y_1} = \frac{y_1-\bar{y}}{\sigma_y}$$

the subscript 1 referring to the volume distribution. This equation can be derived from Eq. 9, by integrating between $-\infty$ and Z instead of between $-\infty$ and $+\infty$, although it was originally proved differently.

Thus

$$Q_1 = M_1 = \frac{1}{\sqrt{2\pi}} e^{\bar{y}+\sigma_y^2/2} \int_{-\infty}^{z} e^{-\frac{1}{2}z^2} dz \quad \text{, where } Z = \frac{y_1-\sigma_y^2-\bar{y}}{\sigma_y} = K_{y_1}-\sigma_y$$

and from Eq. 11

$$e^{\bar{y}+\sigma_y^2/2} = \bar{x}$$

$$\therefore Q_1 = \frac{\bar{x}}{\sqrt{2\pi}} \int_{-\infty}^{K_{y_1}-\sigma_y} e^{-\frac{1}{2}(K_{y_1}-\sigma_y)^2} d(K_{y_1}-\sigma_y)$$

3. References to equations numbers refer to those of the author.

If Q_1 is expressed in terms of the mean $\bar{x}$, then

$$\frac{Q_1}{\bar{x}} = Q_2 = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Ky_1 - \sigma_y} e^{-\frac{1}{2}(Ky_1 - \sigma_y)^2} d(Ky_1 - \sigma_y)$$

shown as the upper line in the writers' Fig. 2.
When the probabilities Q and Q_2 are equal

$$Ky = Ky_1 - \sigma y$$

and hence

$$y_1 - y = \sigma y^2$$

Thus

$$\frac{x_1}{\bar{x}} = e^{\sigma_y^2} = \left(\frac{\bar{x}}{x_g}\right)^2 = C_g^2$$

, which is constant for any given case.
(NOTE: Xg = M in the author's nomenclature. In Fig. 2.C_g^2 has the value 4.5). Hence the two distribution lines are parallel.

A convenient graphical method of locating the volume distribution line is shown in the writers' Fig. 2, based on the relation

$$\frac{x_{g'}}{\bar{x}} = \frac{\bar{x}}{x_g} = C_g$$ which is derived from the previous equation.

It was subsequently discovered that Herdan(1)[4] had utilized and extended this principle in studying the distribution of particles; the parameter x in such cases measures particle size and $\psi(x)$ represents the distributions. If x is a diametral measure, then x^2 is a measure of surface area and x^3 of volume or weight. It may be shown that all the moment distributions $x^r\psi(x)$ are log-normal with the same standard deviation of the logs.
Thus from Eq. 10 -

$$M_r = e^{r\bar{y} + r^2/2\, \sigma_y^2}$$

when r = 0, 1, 2, 3,etc. . . . . . , we obtain

$$M_0 = 1$$

$$M_1 = e^{\bar{y} + \frac{1}{2}\sigma_y^2}$$

$$M_2 = e^{2\bar{y} + 2\sigma_y^2}$$

$$M_3 = e^{3\bar{y} + 9/2\, \sigma_y^2}$$

Taking the ratios of the moments:

$$\frac{M_1}{M_0} = \bar{x} = e^{\bar{y} + \frac{1}{2}\sigma_y^2}$$

4. Reference numbers refer to the writers' bibliography in Appendix II.

$$\frac{M_2}{M_1} = \bar{x}_1 = e^{\bar{y} + 3/2\,\sigma_y^2}$$

$$\frac{M_3}{M_2} = \bar{x}_2 = e^{\bar{y} + 5/2\,\sigma_y^2}$$

Hence the ratio of the means:

$$\frac{\bar{x}_1}{\bar{x}} = \frac{\bar{x}_2}{\bar{x}_1} = \frac{\bar{x}_3}{\bar{x}_2} = e^{\sigma_y^2} = C_g^2 \qquad \text{is constant.}$$

To show that the respective standard deviations are constant we have

$$\sigma_x = \frac{M_2 M_0 - M_1^2}{M_0^2} = e^{\bar{y} + \frac{1}{2}\sigma_y^2}\left(e^{\sigma_y^2} - 1\right)^{\frac{1}{2}} = \bar{x}\, C_v$$

$$\sigma_{x_1} = \frac{M_3 M_1 - M_2^2}{M_1^2} = e^{\bar{y} + 3/2\,\sigma_y^2}\left(e^{\sigma_y^2} - 1\right)^{\frac{1}{2}} = \bar{x}_1 C_v$$

$$\sigma_{x_2} = \frac{M_4 M_2 - M_3^2}{M_2^2} = e^{\bar{y} + 5/2\,\sigma_y^2}\left(e^{\sigma_y^2} - 1\right)^{\frac{1}{2}} = \bar{x}_2 C_v$$

$$\therefore \quad \frac{\sigma x}{\bar{x}} = \frac{\sigma x_1}{\bar{x}_1} = \frac{\sigma x_2}{\bar{x}_2} \quad \cdots = \frac{\sigma x_n}{\bar{x}_n} = C_v$$

and the lines representing the moment distributions are parallel. A more elegant proof is no doubt available.

Relation with Extreme-Value Law

The author gives a very useful discussion on the relation between the extreme-value law and the log-probability law, showing in his Table 3 that, when C_s = 1.139 for the log-normal, the difference between the two distributions is practically negigible. One of the writers (G.N.A.) gave a graphical illustration of this similarity,(2) in the case of maximum annual values of 3-day rainfalls in N.S.W., Australia. As the mean value of σ_{10} (the standard deviation of their logarithms) was 0.178, and hence C_s about 1.3 it was not possible to discriminate on empirical grounds between the log-normal and extreme-value distribution.

It is not clear what the author means by the sentence, "It means that for a given value of Cv = 0.364, the log-probability law furnishes a great number of conditions to fit curves which may be either straight or concave upward or concave downward as it appears on log-probability paper." Perhaps the word "law" in this context is not synonymous with "distribution." On theoretical grounds, as there are two disposable parameters in each case, the log-normal and extreme-value distributions appear equally flexible.

The author seeks to increase the flexibility by removing the relation between C_v and C_s in the log-normal distribution. He gives no examples and as yet the writers have not tried it, but for hydrologic analysis they are rather skeptical of the value of curve-fitting if the third moment has to be used. The author says that his proposed procedure has in reality a combination of the merits of both (i.e. Hazen and Gumbel) methods.

The writers suggest that there is a more rational way of combining the

log-normal and extreme-value distributions, at least for hydrologic analysis. Thus flood flows can be regarded as the extreme-values of daily flow and as daily flows are often close to the log-normal distribution, flood flows in such cases form an extreme-value distribution derived from a log-normal parent. However, the extreme-value distribution used by Gumbel and upon which Powell's paper is based, is the asymptotic case and makes no assumptions concerning the parent distribution; but as the final form is reached so slowly its use in practice appears unrealistic. Dr. E. J. Williams, in a private communication, shows that if the parent distribution is log-normal, the approach is even slower than in the case of normal distribution. For practical purposes, development of what is normally termed small sample theory is required, but in this context the word "small" really means finite.

Bessel's Correction

There are some questions concerning the application of Bessel's correction, i.e. the factor $\frac{n}{n-1}$. This correction has been omitted in the determination of Cv, Cs, etc. in the section on characteristic values, and hence has not entered into the useful relation $Cs = 3\,Cv + Cv^3$. However, in both Eqs. (38) and (39) the author uses the correction $\frac{n}{n-1}$.

Using Fisher's k statistics,[3] we have

$$g_1 = \frac{n}{(n-1)(n-2)} \cdot \frac{\Sigma(x-\bar{x})^3}{\sigma_x^3}$$

or since Eq. (39) can be written

$$Cs = \frac{1}{n-1} \cdot \frac{\Sigma(x-\bar{x})^3}{\sigma_x^3}$$

Hence $g_1 = \frac{n}{n-2}\,Cs$

Hazen also made an adjustment to his Cs values using the factor given by Eq (43).

$$F = 1 + \frac{8.5}{n}$$

It is possible that Hazen's empirical correction may be justified in part by Fisher's extension of Bessel's correction; however as

$$1 + \frac{8.5}{n} > \frac{n}{n-2} \quad \text{if } n > 3$$

Hazen's correction is generally greater.

CORRECTIONS

Page 3, line 17 from top should be variance instead of standard deviation, and in the following line, variances instead of standard deviations.

Page 10, the right side of equation (38) should be under the square root.

In conclusion the writers thank the author for this paper and others he has written which are assisting to bridge the gap between the statistician and the engineer. The need for this liaison is essential in an era of specialization to prevent the marginal field which should be common ground becoming a no-man's land.

Alexander and Karoly on Ven Te Chow

Appendix I. SUGGESTED SYMBOLS

DESCRIPTION	Symbol	
	Used by Author	Suggested by Writers
Median of x, or geometric mean of x	M	x_g
Ratio of arithmetic and geometric means $\frac{\bar{x}}{x_g}$	-	C_g
Log $_e x$	y	x_e
Log $_{10} x$	-	x_{10}
Mean value of log $_e x$	$\bar{y}$	$\bar{x}_e$
Mean value of log $_{10} x$	-	$\bar{x}_{10}$
Standard deviation of x	σ_x	σ
Standard deviation of log $_e x$	σ_y	σ_e
Standard deviation of log $_{10} x$	-	σ_{10}
Logarithm to base 10	$\log_{10}$	log
Logarithm to base e	$\log_e$	ln

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Appendix II. LIST OF REFERENCES

1) "Small Particle Statistics," G. Herdan, p. 118. Elsevier Publishing Co.

2) Discussion of J. F. McIllwraith's paper, "Rainfall Intensity-Frequency Data for N.S.W. Stations," G. N. Alexander, Journal of the Institution of Engineers, Australia, Vol. 26, No. 6, 1954, pp. 119-122.

3) "Statistical Methods for Research Workers," R. A. Fisher. Tenth Edition, 1948. Section 14.

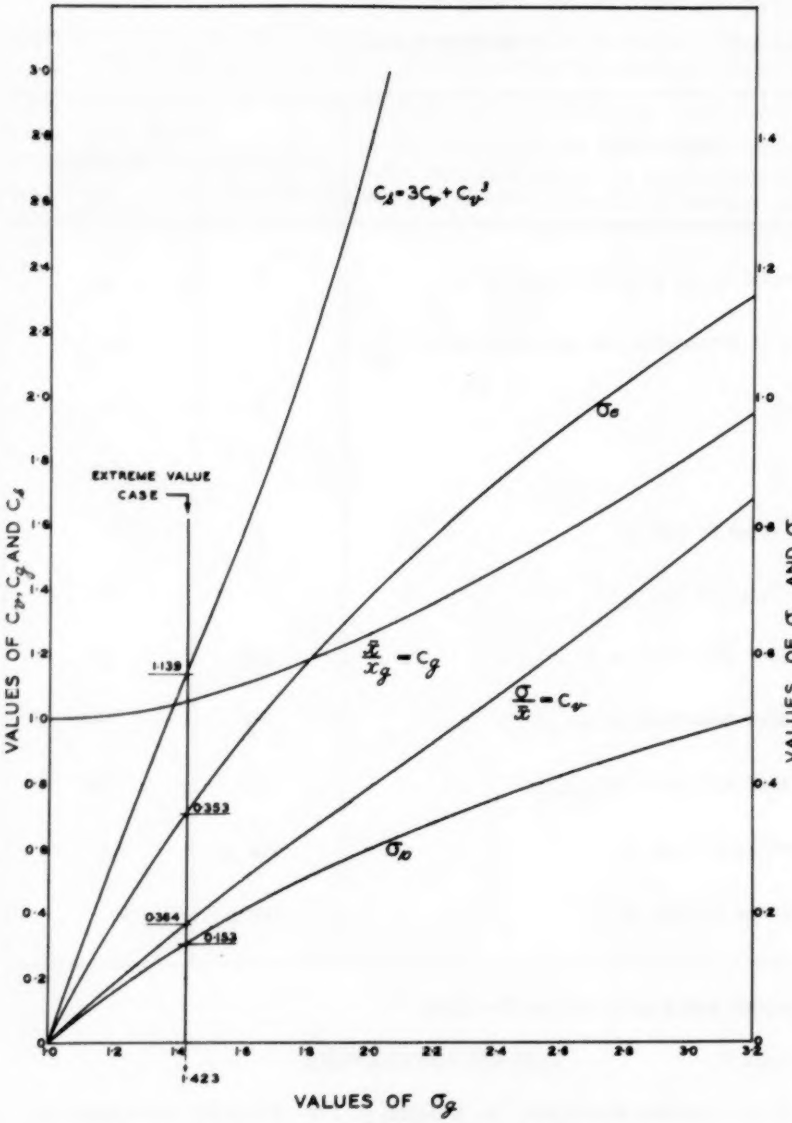

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Fig. 1. Characteristic Values of the Log-Probability Law.

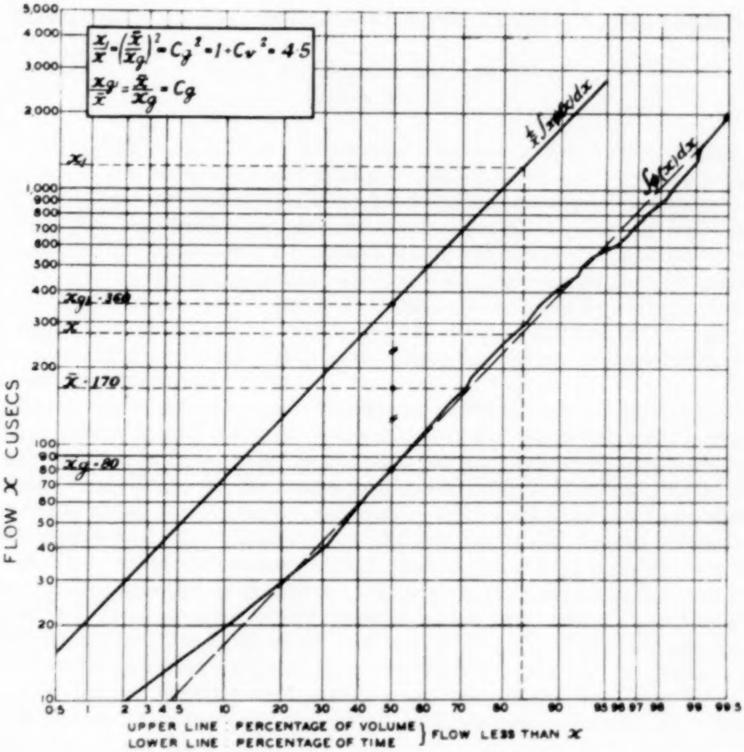

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Fig.2. Derivation of Volume Distribution Curve.
(Delatite River 1947-48 to 1953-54)

Discussion of
"SIMILARITY OF DISTORTED RIVER MODELS WITH MOVABLE BED"

By H. A. Einstein and Ning Chien
(Proc. Sep. 566)

T. BLENCH,[1] M. ASCE.—The authors have discussed river model scales in terms of the knowledge and outlook obtained from studying bed-load transport in flumes; the writer has discussed the same subject in Separate 667 in terms of the knowledge and outlook obtained from studying the self-adjustment of large canals and rivers.(6) Superficially the results must look very different, because of the large number of formulas required by the one method and the few required by the other. Yet, if the mathematics be disregarded so that essential principles can be compared, the authors and the writer appear to have expressed identical views on all important dynamical matters, whether postulated or deduced.

The concordance of essential opinions, arrived at from quite opposite viewpoints, seems to call for some explanation of the superficial differences. The writer prefers to develop an explanation starting from his own viewpoint of regime theory.(1, 2, 3) He thinks of a model-maker as being able to impose only three scales on a mobile-bed model; after they have been imposed all other scales will adjust themselves whether the model-maker wants them to or not. Any three independent scales are imposeable, but for the argument they may be taken as (i) discharge (ii) side-factor (implying tractive force intensity in the side zone where there is no bed-movement), and (iii) bed-factor (implying Froude Number in terms of depth). Obviously the discharge scale can be imposed directly. The side-factor scale can be imposed indirectly by calculating directly-imposeable ones from it and imposing them. The bed-factor scale is imposed by using a suitable bed-material, in suitable quantities, with a fluid of suitable nature; but, as the authors point out, and the writer has also stated in Separate 667, there are circumstances under which only unity scale is permissible.(6) To find how to produce a required bed-factor with a given material requires a couple of weeks of flume experiments.

Suppose now that the model-maker had so vast a knowledge of sediment-transport mechanics that he knew exactly how the bed-factor depends on all its physical constituents. He could then dispense with the couple of weeks of experiment and specify, from his knowledge, that a sand of a (i) definite median size, with (ii) a definite dispersion of grain sizes about the median and (iii) a definite relative density should be used with a certain (iv) charge (meaning ratio of weight per second of material to weight per second of water). Further, the fluid should have a definite (v) viscosity and (vi) suspended load. To be more exact he should also specify angularity of grains, and perhaps the ratio of grain size to water-depth and even the ratio of channel width to depth in cross-over sections; but the writer believes these factors would not be

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particularly important. Thus, for the one item bed-factor he would have at least six, and perhaps nine, which, added to the discharge and side-factor would give eleven degrees of freedom. As the writer sees the matter, the authors have specified most of these constituent parts of bed-factor, so have had to produce equations to correspond. The actual identification of the factors from the equations is difficult without some practice, and there seems no point in attempting it here.

Whether the authors' various equations are accurate is another matter, and they do not appear to claim that they are. The writer feels there are two points concerning them that deserve comment. The first is that the equations rest on a prior elimination of the effect of sides by a method he thinks is dubious. The second is that, according to regime theory, equation (1) contains D in an incorrect manner, but m ought to be 1/4. The incorrectness arises from the regime theory finding (from observed behavior of channels in the field) that the linear dimension characterising roughness should not be D, but a composite of the shear stress on the sides and the bed-factor; D is implicit in this, but would appear in the numerator instead of the denominator if made explicit—a paradox that used to worry river engineers till they found it was associated with dune movement. The writer has commented, in his discussion of Separate 611, that extrapolation of the authors' formulas to design of large channels is not satisfactory; perhaps equation (1) is a principal cause, but recent regime analysis of flume data[4, 5] seems to show that no existing information possesses the accuracy, range, or completeness to justify extrapolation beyond a very low limit.

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